THE MATHEMATICS EFFECTIVENESS IN ECONOMICS

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Abstract: *The question this paper tries to give some answers concerns the important role of mathematics in the world of economics. The answer that the author has found is to underline the influence of the mathematical background in the life of some of the most important economists of our and all days, the Nobel laureates. This article wants to be a survey of the list of the winners of the Nobel Prize in Economic Sciences who have declared their mathematical approach or studies.*
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1. Introduction

Economic sciences in recent years have experienced great prosperity, especially in terms of the number of students who choose this field of study. Being so popular field, selection of candidates is not very restrictive and the consequence is the question increasingly more common, even among economists, if mathematical background is or is not a necessity for a future economist. Especially considering that the economy is a field as discussed in everyday life so that her scientific character are almost forgotten.

We can discuss about economy at various levels. First, in terms of running a small business, where you can do without the

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Lagrange multiplier method, or more scientific level, of world-class economic theories where basic knowledge are more complex. At the highest scientific level, best reference points can only be Nobel Prize winners. And if we analyze their mathematical background, we have an indication of the usefulness of mathematical knowledge accumulation, i.e., a clue about of the mathematics effectiveness in economics.

2. How a Nobel Prize in Economics is actually awarded

In 1968, Sveriges Riksbank (the Central Bank of Sweden) instituted a new award, "The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel" on the basis of an economic commitment by the bank in perpetuity. The award is given by the Royal Swedish Academy of Sciences according to the same principles as for the Nobel Prizes that have been awarded since 1901.

The procedures for selecting the Laureates are also the same. Each year the Academy receives some 200-300 nominations, usually covering a little more than one hundred nominees. (Unsolicited suggestions from persons who have not been asked to submit nominations are not considered.) The Economics Prize Selection Committee of the Academy (with five to eight members) commissions expert studies of the most prominent candidates, sometimes by Swedish experts, but usually by foreigners. The Prize Committee presents its award proposal to the Social Science Class of the Academy (Class IX) in the form of a report, with an extensive survey of the main candidates that are considered for a Prize. The report motivates the proposal and includes all the solicited expert studies. On the basis of this material the class suggests a Laureate (or a shared Prize between two or, at most, three Laureates) regularly following the committee's proposal. Finally the entire Academy meets to take the final award decision, usually in October.

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel has been awarded 45 times to 74 Laureates between 1969 and 2013. 22 Prizes in Economic Sciences have been given to

one Laureate only, 17 Prizes in Economic Sciences have been shared by two Laureates and 6 Prizes in Economic Sciences have been shared between three Laureates. The average age of all Laureates in Economic Sciences between 1969 and 2012 is 67 years. To date, the youngest Laureate in Economic Sciences is Kenneth J. Arrow, who was 51 years old when he was awarded in 1972. The oldest Laureate in Economic Sciences to date is Leonid Hurwicz, who was 90 years old when he was awarded in 2007. He is also the oldest Laureate to be awarded in all Prize categories (http://www.nobelprize.org).

3. Mathematical background for the special people who influence the world of economics: the Nobel Laureates in Economic Sciences

Starting with the beautiful "Ideological profiles of the Economics Laureates" (Klein *et al.*, 2013) in this section we provide a brief view on one particular list of the Nobel Laureates in Economic Sciences in order to underline the role of mathematics in forming and developing their brilliant minds. This list is in alphabetical order and contains forty two names from the total of seventy four. Perhaps the list could be even longer, because we are certain that some (or could be almost all) of the laureates that we did not include here, have strong knowledge of mathematics, but they did not insist to recall it, in their curriculum vitae.

Maurice Allais (1911–2010) began his career in *engineering* but following the Great Depression, he turned to economics. Maurice Allais said in his Nobel lecture from 1988: "In July 1940, after my demobilization, I resumed again my duties as an engineer in the State mining administration at Nantes, in the zone occupied by the German Army. But my pre-war concerns had completely changed. For me, with all the illusions of youth (I was only 29 at the time), it was clear that the best I could do was to contribute to prepare for the postwar period". He became a professor of economics at the Ecolè National Superieure des Mines in 1944 and remained in that position thereafter. His work on capital and interest rates would assist French planners in developing long-term economic strategies (Klein *et al.*, 2013).

Maurice Allais won the Nobel Prize, in 1988, for his "pioneering contributions to the theory of markets and efficient utilization of resources".

Kenneth J. Arrow (1921–), first, taking *mathematics* as major, he attempted to pursue a career as a high school teacher. When he found that market flooded, he went to Columbia University to study statistics and discovered the economics professor Harold Hotelling. Taking Hotelling's course on mathematical economics,

Arrow realized that he had found "his niche", as he said himself. At Columbia University, he earned his M.A. in mathematics and entered the Ph.D. program in economics, but World War II put his studies on hold. Returning to civilian life after war, he continued his graduate work. He completed his Ph.D. in 1951. His enormous impact on economic life is suggested by the number of major ideas that bear his name: Arrow's Theorem, the Arrow-Debreu model, the Arrow-Pratt index of risk aversion, and Arrow securities" (Starr, 2008). Besides the four areas alluded to in the quotation from Starr, Arrow has been a leader in the economics of information.

In 1972, at the age of 51 (still the youngest ever), Arrow shared the Nobel Prize in Economic Sciences with John Hicks for their "contributions to general economic equilibrium theory and welfare theory" (Klein *et al.*, 2013).

Robert Aumann (1930–) attended Rabbi Jacob Joseph School in New York City where he became acquainted with geometry, theorems, and proofs and decided to pursue *mathematical studies* over Talmudic studies. He earned his bachelor's degree from City College of New York. He went on to the Massachusetts Institute of Technology for his Ph.D. in algebraic topology in 1955. After working as a strategic consultant and two years as a post-doc at Princeton University, he went to Israel in 1956 to teach at Hebrew University in Jerusalem. He is one of the founding members of the Center for Game Theory in Economics at Stony Brook University.

He shared the Nobel Prize with Thomas Schelling for "having enhanced our understanding of conflict and cooperation through game-theory analysis". The most strongly cited for Aumann's award is

"the strong equilibrium payoffs of a repeated game coincide with the core (more precisely, β-core) payoffs in the one-shot game" - the idea is that cooperative outcomes can be sustained in games with repeated play (Klein *et al.*, 2013).

Gary Becker (1930–) was born in a coal mining town in eastern Pennsylvania, but the family moved to Brooklyn when Becker was four or five years old. Becker's parents had only received an 8th grade education. Becker reflected: "There were only a few books in our house, but my father kept up with the political and financial news, and my older sister read a lot. After my father lost most of his sight, I had the task of reading him stock quotations and other reports on financial developments. Perhaps that stimulated my interest in economics, although I was rather bored by it" (Becker, 1993). He recalls "we had many lively discussions in the house about politics and justice. I believe this does help explain why by the time I finished high school, *my interest in mathematics* was beginning to compete with a desire to do something useful for society. These two interests came together during my freshman year at Princeton, when I accidentally took a course in economics, and was greatly attracted by the mathematical rigor of a subject that dealt with social organization" (Becker, 1993). He also said "in the first course I took in economics as a freshman at Princeton, we had a textbook—and that was Paul Samuelson's. It was fascinating. What impressed me about that famous textbook was that in the last quarter of that book…he had a mathematical formulation of economics. That was the part of the whole course that really attracted me the most. To me the question was how I could use mathematics - which I liked and was pretty good at - to discuss social questions. So Samuelson's book actually built the bridge" (Becker, 2009).

Gary Becker won the Nobel Prize in Economic Sciences in 1992 for "expanding microeconomic analysis to non-market behavior such as discrimination, human capital, the family, demography, criminology, and addictive behavior" (Klein *et al.*, 2013).

Gerard Debreu (1921–2004) was born in Calais. He attended the college there, where he was introduced to the "*austere beauty of mathematics*". At school, Debreu came under the influence of

Henri Cartan, a founding member of the Bourbaki collective of mathematicians. Till Duppe describes the collective and Debreu's connection to it: "The most striking feature that Debreu must have experienced as liberating was the anonymity of mathematics that 'Bourbaki' as a collective represented. Rather than each single member speaking out in his own name, they aimed at letting *mathematics speak for itself*" (Duppe, 2012).

Well known for his collaboration with Kenneth Arrow (1954) to produce a general equilibrium proof, Debreu focused on the mathematics, not on ideology of economics. Duppe wrote that Debreu "was silent about the use of general equilibrium theory for either social engineering or libertarian justification" (Duppe, 2012). The Prize in Economic Sciences was awarded to Gerard Debreu in 1983 "for having incorporated new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium"(Klein *et al.*, 2013).

Peter Diamond (1940–) graduated *summa cum laude* from Yale University with a major in *mathematics*. At Yale, he took several economics courses, including one taught by Gerard Debreu. After his undergraduate education, during the summer of 1960, he worked under Tjalling Koopmans surveillance at the Cowles Commission. He decided to pursue his graduate degree, studying both math and economics at MIT before switching completely to economics. After earning his Ph.D. in economics, Diamond taught at the University of California at Berkeley for several years before returning to MIT, where he has been ever since (Klein *et al.*, 2013).

In 2010, he shared the Nobel Prize in Economics with Dale Mortensen and Christopher Pissarides for their "analysis of markets with search frictions".

Robert Engle (1942–) started on a Ph.D. in physics at Cornell University but switched to economics after his first year. Despite only having had one economics course in his undergraduate education, Engle became increasingly interested in social science: "I was intrigued by the notion of applying myself to the most quantitative social science. This would allow me to *use my mathematics* and yet still

study the interesting problems of modern mankind. It seemed to me that economics could be the path to esoteric academic modeling or to solving practical real world problems" (Engle, 2003). After earning his Ph.D degree, Engle joined the Massachusetts Institute of Technology, later he moved to the University of California at San Diego, and then to New York University (Klein *et al.*, 2013).

Engle was awarded the Nobel Prize in Economics in 2003, along with Clive W. J. Granger, "for methods of analyzing economic time series with time-varying volatility (ARCH)". Engle and Granger won for their joint work on cointegration and developing a model to deal with heteroskedastic volatility in data with nonstationary means, work which has "completely revolutionized the field of time series econometrics and the practice of empirical macroeconomics and asset pricing finance" (Bollerslev, 2008). Kevin Hoover says that "Granger and Engle coined the term 'cointegration' to describe the genuine relationship between two nonstationary time series. Time series are 'cointegrated' when the difference between them is itself stationary" (Hoover, 2008). Realizing the applicability of ARCH to finance, Engle has focused largely on finance (Bollerslev, 2008).

Milton Friedman (1912–2006) enrolled at Rutgers University as a 16 year-old in 1928, and graduated in 1932, where he *specialized in mathematics* and initially intended *to become an actuary*. After graduating from Rutgers, Friedman was offered two scholarships to do graduate work: one in mathematics at Brown University and the other in economics at the University of Chicago. He chose the latter, thus earning an M.A. in 1933.

In 1976, Friedman won the Nobel Prize in Economics for his "achievements in the fields of consumption analysis, monetary history and theory and for his demonstration of the complexity of stabilization policy" (Klein *et al.*, 2013).

Ragnar Frisch (1895–1973), when he went to the university, he chose political economy because, according to himself, that was the "*shortest* and *easiest* study" at the University of Oslo. He graduated with distinction from the Royal Fredericks University of Oslo in 1919, with the degree Cand. oecon. in political economy. To complete his

studies he was rewarded a fellowship from the university and went abroad *to study mathematics, statistics* and economics. He stayed nearly three years in France, but visited also Germany, Great Britain and Italy. On his return to the university he continued his scientific activity believing that research was his calling. In 1925 he became an assistant on a research program in production theory led by Professor Petter Thorvald Aarum (1867–1926). In 1926 he defended his doctoral thesis, "Sur les semi-invariants et moments employes dans l'etude des distributions statistiques," a work on time series and statistics, at the Faculty of Mathematical and Natural Sciences. In 1927 Frisch received a fellowship from the Rockefeller Foundation and went to the United States where he met the leading mathematical economists of the day, among them Irving Fisher, Wesley Clair Mitchell, Henry Schultz, and Allyn Young. Frisch, Fisher, and Charles Roos began planning the formation of an association that came to be the Econometric Society. In 1929 he started to lecture as Associate Professor on the theory of production, in which he extensively used mathematics.

When the Swedish Central Bank established its Prize in Economic Sciences in 1969, the prize was awarded jointly to Ragnar Frisch and Jan Tinbergen for "having developed and applied dynamic models for the analysis of economic processes" (Klein *et al.*, 2013).

Clive William John Granger (1934–2009) applied to the University of Nottingham for its first intake of a joint degree program in mathematics and economics. Granger claimed that his two first-year courses in economics were the only formal training he ever received in economics. He *switched to mathematics fulltime* and obtained his First in the subject. Granger then earned his Ph.D. in statistics in 1959 and spent a year at Princeton working on Oskar Morgenstern's Time Series Project. He became a junior lecturer in statistics in 1956 and was on the Nottingham faculty for two decades. In 1974 he moved to the University of California at San Diego.

Granger together with Robert Engle won the 2003 Nobel Prize in Economics "for methods of analyzing economic time series with common trends (cointegration)" (Klein *et al.*, 2013).

Trygve Magnus Haavelmo (1911–1999) was an influent economist with main research interests centered on the fields of econometrics and economics theory. He received a degree in economics from the University of Oslo in 1930 and eventually joined the Institute of Economics with the recommendation of Ragnar Frisch. He was Frisch's assistant for a period of time, until he was appointed as head of computations for the institute. In 1936, he studied statistics at University College of London while he subsequently traveled to Berlin, Geneva, and Oxford for additional studies. Trygve Haavelmo assumed a lecturing position at the University of Aarhus in 1938 for one year and then in the subsequent year was offered an academic scholarship to travel abroad and study in the United States. During World War II he worked with Nortraship in the Statistical Department in New York City. He received his Ph.D. in 1946 for his work on "The Probability Approach in Econometrics".

In 1989, Trygve Haavelmo was awarded the Nobel Prize in Economics "for his clarification of the probability theory foundations of econometrics and his analyses of simultaneous economic structures" (http://en.wikipedia.org).

Lars Peter Hansen (1952–), after graduating from Utah State University, B.S. in mathematics and political science, in 1974, and the University of Minnesota, Ph.D. in economics in 1978, he served as assistant and associate professor at Carnegie Mellon University before moving to the University of Chicago in 1981. He is currently the David Rockefeller Distinguished Service Professor in Economics at the University of Chicago.

In 2013, he was awarded the Nobel Prize in Economics, jointly with Robert J. Shiller and Eugene Fama, "for their empirical analysis of asset prices" (http://en.wikipedia.org).

John Harsanyi (1920–2000) graduated from the Lutheran Gymnasium in Budapest, alumni of which include John von Neumann. In 1937, the year that Harsanyi graduated, he won *the First Prize in Mathematics* at a nationwide competition for high school students. Harsanyi earned his Ph.D. in philosophy in 1947, and long after he earned a master's in economics from the University of Sydney, which he

completed in 1953. He earned the Rockefeller Fellowship in 1956, which allowed him to come to the United States and earn his Ph.D. at Stanford. There, Harsanyi began working with his dissertation advisor, Kenneth Arrow, who recommended that he study mathematics and statistics.

John Nash's work on cooperative and noncooperative games inspired Harsanyi to begin mathematically analyzing game theory. Harsanyi developed Nash's formulations to include asymmetric information, such as in negotiations (Haas, 1994). When Harsanyi worked with nine other game theorists to advise the United States in negotiations with the Soviet Union, he said, "We discovered that we couldn't advise them on this matter because these negotiations represented a game with incomplete information, in which each side knew little about the other side" (quoted in Haas, 1994). Harsanyi proceeded in later years to develop game theory with an eye toward making it more effective in real-world situations. He won the Nobel Prize in 1994 along with Reinhard Selten and John Nash "for their pioneering analysis of equilibria in the theory of non-cooperative games" (Klein *et al.*, 2013).

James Heckman (1944–) received his B.A. *in mathematics* from Colorado College and received his Ph.D. from Princeton University in economics in 1971. Heckman then served as an Assistant Professor at Columbia University before moving to the University of Chicago in 1973. In addition to serving as the Henry B. Schultz Distinguished Service Professor of Economics, Heckman is also the director of the Economics Research Center and the Center for Social Program Evaluation at the Irving B. Harris School of Public Policy. In 2004, he was appointed as the Distinguished Chair of Microeconometrics at University College London. In June 2006 he was appointed as the Professor of Science and Society at University College Dublin. Heckman is also a senior research fellow at the American bar Foundation.

Heckman shared the Nobel Prize in Economics in 2000 with Daniel McFadden for his "pioneering work in econometrics and microeconomics" (http://en.wikipedia.org).

John Richard Hicks (1904–1989), financed by mathematical scholarships, he went to Clifton College between 1917 and 1922 and Balliol College, Oxford between 1922 and 1926 for mathematics. Hicks

reflected on his move towards economics: "[D]uring my school days, and in my first year at Oxford, I was a mathematical specialist; to the mathematical training I received at Clifton, in particular, I owe a great debt. But I was not contented with mathematics; I had interests in literature and in history which I needed to satisfy. My move (in 1923) to "Philosophy, Politics and Economics," the "new school" just being started at Oxford, was, however, not a success. I finished with a second-class degree, and no adequate qualification in any of the subjects I had studied. Economists, in those days, were very scarce, so I did pick up a temporary lecturership at the London School of Economics and managed to get continued. I started as a labour economist, doing descriptive work on industrial relations, but, gradually, I moved over to the analytical side. Then I found that *my mathematics*, by that time almost forgotten, *could be revived*, and were sufficient to cope with what anyone (then) used in economics" (Hicks, 1992/1973).

Hicks won the Nobel Prize in Economic Sciences in 1972 with Kenneth J. Arrow for "their pioneering contributions to general economic equilibrium theory and welfare theory" (Klein *et al.*, 2013).

Leonid Hurwicz (1917–2008) encouraged by his father to study law, he received in 1938 his LL.M. degree from the University of Warsaw, where he discovered his future vocation in economics class. He then studied at the London School of Economics with Nicholas Kaldor and Friedrich Hayek. In 1939 he moved to Geneva where he studied at the Graduate Institute of International Studies and attended the seminar of Ludwig von Mises. After moving to the United States he continued his studies at Harvard University and the University of Chicago. Hurwicz had no degree in economics but he is considered both economist and mathematician. He taught subjects ranging from theory to welfare economics, public economics, mechanisms and institutions and mathematical economics. In 2007 he said, "Whatever economics I learned I learned by listening and learning."

He originated incentive compatibility and mechanism design, which show how desired outcomes are achieved in economics, social and political sciences. Interactions of individuals and institutions, markets and trade are analyzed and understood today using the models

Hurwicz had developed. Leonid Hurwicz is the oldest Nobel Laureate, having received the prize at the age of 90.

Hurwicz was Regents' Professor of Economics (Emeritus) at the University of Minnesota. He was among the first economists to recognize the value of game theory and was a pioneer in its application. Hurwicz shared the 2007 Nobel Memorial Prize in Economic Sciences with Eric Maskin and Roger Myerson for "their work on mechanism design" (http://en.wikipedia.org).

Leonid V. Kantorovich (1912–1986) had a *talent for mathematics* and finished high school early at age 14 to enter Leningrad University. He graduated in 1930, and then received a professorship in 1934 and finally his Ph.D in mathematics in 1935, all by the age of 23. Kantorovich began his career as a mathematics professor, but forayed into economics in the late 1930s, when he began working on complex problems of resource allocation (Kantorovich, 1992/1976). Aron Katsenelinboigen says about Kantorovich: "In 1939–41, he realized that the socialist economy as a whole could be perceived as an optimization problem. The logic of the Soviet planned economy naturally impelled Kantorovich towards this notion. ... In investigating [optimization relations], he was also able to penetrate deeper into the role of prices than had Soviet economists before him" (Katsenelinboigen, 1979).

In 1975, Kantorovich won the Nobel Prize, shared with Tjalling Koopmans, for "contributions to the theory of optimum allocation of resources" (Klein *et al.*, 2013).

Lawrence Robert Klein (1920–2013) describes his direction in college: "When I entered university my interests began to take shape in the world of ideas. I specialized in both economics and mathematics. I could not see their eventual use together to deal with problems that the world faced. Also, I was not equal to the quick-witted star mathematicians at the university, but I kept being *attracted by mathematical problems* and their potential use in natural, physical, and social sciences - especially in economics" (Klein, 2006).

For his work in creating computer models to forecast economic trends in the field of econometrics at the Wharton School of the University of Pennsylvania, he was awarded the Nobel Prize in

Economic Sciences in 1980 specifically "for the creation of econometric models and their application to the analysis of economic fluctuations and economic policies." Due to his efforts, such models have become widespread among economists. Harvard University professor Martin Feldstein told the Wall Street Journal that Klein "was the first to create the statistical models that embodied Keynesian economics," tools still used by the Federal Reserve Bank and other central banks (http://en.wikipedia.org).

Tjalling Charles Koopmans (1910–1985) was a Dutch-American mathematician and economist, the joint winner with Leonid Kantorovich of the 1975 Nobel Prize in Economic Sciences for "contributions to the theory of optimum allocation of resources". He began his university education at the Utrecht University at seventeen years, specializing in mathematics. Three years later, in 1930, he switched to theoretical physics. In 1933, he met Jan Tinbergen, the 1969 Bank of Sweden prize winner, and moved to Amsterdam to study mathematical economics under his supervision. In addition to mathematical economics, Koopmans extended his explorations to econometrics and statistics. In 1936 he graduated from Leiden University with a Ph.D. The title of the thesis was "Linear regression analysis of economic time series" (http://en.wikipedia.org).

Finn E. Kydland (1943–) concentrated on math and physics in high school. He has said "I knew more math at the end of high school than a typical American business or economics major" (Kydland, 2005). He continued to the Norwegian School of Economics and Business Administration [NHH], in Bergen. Kydland has explained why he chose that school, rather than the University of Oslo, which had greater prominence in academic economics: "When I started studying at NHH, it was not for the sake of studying economics. An acquaintance of mine...had set up a couple of innovative businesses...and he introduced me to business. I helped him with his accounting and soon realized I would rather be a business economist...than a civil engineer..., on which I had set my sights originally. So when I arrived at NHH I had no idea what modern economics was about. I thought I was studying to get qualifications

to work in business. That I did not end up in business was basically an accident" (Kydland, 2006).

Kydland and Prescott shared the 2004 Nobel Prize in Economics "for their contributions to dynamic macroeconomics: the time consistency of economic policy and the driving forces behind business cycles" (Klein *et al.*, 2013).

Harry Markowitz (1927–) went to the University of Chicago for undergraduate and graduate school, and, interested in questions about utility, he was drawn to economics. His dissertation was about finding solutions on "how to compute efficient sets for large numbers of securities, and how to incorporate mean-variance analysis into the theory of rational behavior under uncertainty." Markowitz recalled Milton Friedman saying, "I've read your dissertation and can't find any mistakes in it. There is just one problem: this is not a dissertation in economics. We cannot award you a Ph.D. in economics for a dissertation that is not economics." Markowitz did earn his Ph.D that day (Markowitz, 1991/1959). In Chicago, Markowitz became involved in the Cowles Commission, and at Jacob Marschak's suggestion, worked on *applying mathematics to the stock market* (Markowitz, 2013). Markowitz joined the RAND Corporation in 1952. He then became professor at Baruch College in New York before joining the University of California at San Diego.

He is a recipient of the 1990 Nobel Prize in Economic Sciences, for "his pioneering work in modern portfolio theory, studying the effects of asset risk, return, correlation and diversification on probable investment portfolio returns" (Klein *et al.*, 2013).

Eric Maskin (1950–), in high school, found *an affinity for mathematics*. He attended Harvard University and earned a bachelor's degree in math. During his undergraduate education he happened upon Kenneth Arrow's class on information economics, "which was so inspiring that I decided to change direction. It seemed to me that economics combined the best of both worlds: the rigor of mathematics with the immediate relevance of a social science" (Maskin, 2013). He continued at Harvard for his master's and Ph.D. in applied mathematics, but focused on economics. Under Arrow's direction, Maskin wrote his

dissertation on mechanism design and social welfare (Maskin, 2008). As in his dissertation, Maskin's early work focused on mechanism design implications for social choice and later he worked on auctions, monopoly, and game theory (especially in discontinuous games).

Maskin shared the 2007 Nobel Prize in Economics with Leonid Hurwicz and Roger Myerson for "mechanism design theory" (Klein *et al.*, 2013).

Daniel McFadden (1937–) earned his bachelor's degree in Physics at the University of Minnesota and also became *interested in mathematical modeling of economic choices*. In 2000 interview, McFadden discussed how he came to study economics: "Well, I came from the natural sciences, from physics. I was a graduate student in physics and found psychology very interesting, and the psychology of behavior. I had an opportunity to enter this disciplinary program in the behavioral sciences. And I pursued that for my Ph.D. and got into economics pretty much by accident, because the formal modeling, the axiomatic work that I was interested in was being done primarily by two economists, John Chipman and Leo Hurwicz. So I made economics my specialty in order to work with them (Heckman and McFadden, 2000).

He earned the 2000 Nobel Prize "for his development of theory and methods for analyzing discrete choice," sharing the prize with James Heckman. A major contribution came about when a graduate student sought his help modeling freeway design choices, which led him to build "an econometric model based on an axiomatic theory of choice behavior". The applicability of his ideas is broad: These developments, now called the multinomial logit model and the random utility model for choice behavior, have turned out to be widely useful in economics and other social sciences. They are used, for example, to study travel modes, choice of occupation, brand of automobile purchase, and decisions on marriage and number of children (McFadden, 2001).

David Henderson describes the importance of McFadden's innovation: "Before McFadden's work, empirical economists who studied various issues tended to assume that the variables they were

studying were continuous. This works well when one studies, say, the demand for sugar, because people buy various amounts of sugar along a continuum. But what if one is studying the demand for refrigerators? Because most people have only one refrigerator, the choice of a refrigerator is a discrete choice", (Henderson, 2008).

Robert C. Merton (1944–) earned his B.S. in engineering mathematics. He then moved to the California Institute of Technology for his M.S. in applied mathematics. After his master, he was drawn to economics. He says he believed macroeconomics had been effective in controlling unemployment and inflation, and he "felt that working in economics could 'really matter' and that potentially one could affect millions of people". Merton is very reserved in policy discourse and holds a middle ground, recognizing the potential for both market and political failures. Knowing that his mathematical and engineering background would help him in economic analysis, Merton also felt he had strong economic intuition, particularly regarding the stock market (Merton, 1998).

Robert C. Merton and Myron S. Scholes have earned the Nobel Prize in 1997, for "developing a pioneering formula for the valuation of stock options....Robert Merton devised another method to derive the [Black-Scholes] formula that turned out to have very wide applicability; he also generalized the formula in many directions" (Klein *et al.*, 2013).

James Mirrlees' (1936–) interest and progress in mathematics led him to a mathematics degree at Edinburgh University. He was awarded a grant to pursue another undergraduate degree, Part II of the Cambridge Mathematics Tripos at Trinity College in 1957. By the time he completed Part III with distinction, making him eligible for research in mathematics, he realized he wanted to pursue economics "because poverty in what were then called the underdeveloped countries, seemed to me what really mattered in the world" (Mirrlees, 1997). Drawing on his mathematical background, Mirrlees created a model for solving the problem of optimal taxation. His work yielded at least one surprising result: "I must confess that I had expected the rigourous analysis of income taxation in the utilitarian manner to provide arguments for high tax rates. It has not done so" (Mirrlees, 1971).

In 1996, he was awarded the Nobel Prize, shared with William Vickrey, "for their fundamental contributions to the economic theory of incentives under asymmetric information" (Klein *et al.*, 2013).

Dale Mortensen (1939–2014) went to Willamette University, studying economics and mathematics to "*combine my interest in analytical problem solving with a concern for social issues*" (Mortensen, 2011). Mortensen's Nobel-winning work applied search theory to labor markets and unemployment. Upon receiving the prize, Mortensen summarized his research in a sentence: "It takes time for workers to find jobs and for employers to find workers" (quoted in Tremmel, 2010).

He was awarded the Nobel Prize in Economics jointly with Christopher A. Pissarides and Peter A. Diamond "for their analysis of markets with search frictions"(Klein *et al.*, 2013).

Roger Myerson (1951–) attended Harvard University, where he received his A.B., summa cum laude, and S.M. in *applied mathematics* in 1973. He completed his Ph.D. in applied mathematics from Harvard University in 1976. His Ph.D thesis was *A Theory of Cooperative Games*.

Myerson won the 2007 Nobel Prize in Economics with Eric Maskin and Leonid Hurwicz "for having laid the foundations of mechanism design theory" (Klein *et al.*, 2013).

John Forbes Nash, Jr. (1928–) is an genius mathematician, whose works in game theory, differential geometry and partial differential equations have provided insight into the factors that govern chance and events inside complex systems in daily life. His theories are used in market economics, computing, evolutionary biology, artificial intelligence, accounting, politics and military theory. Serving as a Senior Research Mathematician at Princeton University during the latter part of his life, he shared the 1994 Nobel Prize in Economic Sciences with game theorists Reinhard Selten and John Harsanyi. Nash was credited with introducing "the distinction between cooperative games, in which binding agreements can be made, and non-cooperative games, where binding agreements are not feasible. Nash developed an equilibrium concept for non-cooperative games that later came to be called Nash equilibrium" (http://en.wikipedia.org).

Nash began to study mathematics in high school. Nash states that he was not the "best student" and was voted by his peers as the "most original" student. On originality, he states: "So there is something of that in my approach to mathematics. I have tended to think that the thing to do is to get away from what other people are doing and not to follow directly in anyone's recent work. I did that in game theory also. I brought in some alternative ideas, which, to some extent, are competitive" (Nash, 2002). During his last year of high school he took supplementary mathematics courses at the Bluefield College. On learning during his childhood, Nash states: "I did have an appreciation of math and science as a child, and even at elementary school I would like to do more in mathematics than the other students were doing in one way or another. So I did have that at an early stage. Like *maybe almost comparable to Mozart in music*" (Nash, 2004).

Christopher Pissarides (1948–) discussed his decision to study economics: "Like many well-off Cypriots, I went to London to study for a degree in economics. I wanted to study architecture, but because of my father's business my parents persuaded me to try economics or accounting. *My love at school was mathematics*, but it was not considered to be a good profession for a young man, with which I agreed. When I tried economics I liked it, so I decided to pursue my studies in it" (Pissarides, 2011).

Pissarides shared the Nobel Prize in 2010 with Peter Diamond and Dale Mortensen "for their analysis of markets with search frictions" (Klein *et al.*, 2013).

Edward Prescott' (1940–) *mathematical inclinations* and lessons in business from his father prepared him to be an economist. A mathematical course in engineering introduced Prescott to the new field of operations research. He decided to go to Case Institute of Technology for graduate studies where he "learned a set of tools that proved invaluable in helping to make macroeconomics part of economics." With his new fondness for engineering, Prescott decided to get his Ph.D. at the Graduate School of Industrial Administration at Carnegie Institute of Technology, a multidisciplinary program, which introduced him to economics (Prescott, 2009).

Prescott shared the 2004 Nobel Prize in economics with his collaborator Finn E. Kydland "for their contributions to dynamic macroeconomics: the time consistency of economic policy and the driving forces behind business cycles." Their analysis suggested a need for pre-commitment devices in policy decisions (Kydland and Prescott, 1977).

Alvin Roth (1951–), after taking a few engineering classes at Columbia University, he applied and became a full-time engineering student. He went on to Stanford University for his Ph.D. in *operations research*. Led to engineering, and specifically operations research, "with the idea of fixing things up so they work better," Roth moved into game theory and from game theory into economics (Boyeaux, 2012).

Alvin Roth and Lloyd Shapley shared the 2012 Nobel Prize in Economics "for the theory of stable allocations and the practice of market design" (Klein *et al.*, 2013).

Myron Scholes (1941–), in high school, "enjoyed puzzles and financial issues," *succeeded in mathematics*, physics, and biology, and subsequently was solicited to enter an engineering program by McMaster University. He credits his mother with making him "become interested in economics and, in particular, finance". Working as a computer programmer at the University of Chicago, Scholes assisted professors in developing research design. Some of his first clients were Merton Miller and Eugene Fama. "They became my mentors, shaping my understanding of finance. And, fortunately, both became lifelong friends" (Scholes, 2009). After earning his Ph.D., Scholes taught at MIT, where he met and began collaborating with Fischer Black on financial economics. A little over a decade later, he returned to the University of Chicago, where he focused on taxation. Since 1983 he has been on the faculty at Stanford University. Early on in his life he became fascinated with the stock market, and this interest would lead to his Nobel Prize-winning work (Scholes, 1998).

Along with Robert C. Merton, Scholes won the 1997 Nobel Prize "for a new method to determine the value of derivatives." Most of his work is technical, but Scholes has made policy related comments in interviews and debates, and he seems mostly to favor free markets (Klein *et al.*, 2013).

Reinhard Selten (1930–) had *a knack for mathematics*, something he noticed in high school. From the age of 15 he taught himself mathematics. However, he knew becoming a mathematician was not in his future (Selten, 2009). After high school, Selten received his degree in mathematics at Frankfurt University but switched to economics and began working with Heinz Sauermann. Selten also collaborated extensively with John Harsanyi, particularly on games involving incomplete information. Selten developed the idea of trembling hand perfection, in which equilibria hold despite a small chance of perturbation (Damme, 2008).

Reinhard Selten won the Nobel Prize in 1994, along with Harsanyi and John Nash, "for their pioneering analysis of equilibria in the theory of non-cooperative games" (Klein *et al.*, 2013).

Lloyd Shapley (1923–), as a student at Harvard University, was drafted in 1943 to serve in World War II. As a sergeant in the Army Air Corps in China he earned a Bronze Star for helping *crack the Japanese weather code* (Ferguson, 1991). After the war, he went to Harvard University where he earned his bachelor's degree, and then to Princeton University for his Ph.D. in mathematics.

In 2012, Shapley shared the Nobel Prize in Economics with Alvin Roth "for the theory of stable allocations and the practice of market design." In presenting the Nobel Prize, Torsten Persson (2012) told Shapley: "Your contributions to cooperative game theory are legendary among game theorists and economists. You and David Gale are the founders of matching theory, and the deferred-acceptance algorithm you discovered is the cornerstone on which theory and applications rest" (Klein *et al.*, 2013).

Herbert Simon (1916–2001), in 1933, Simon entered the University of Chicago, and studied the social sciences and *mathematics*. He was interested in biology, but chose not to study it, but instead he focused on political science and economics. His most important mentor at the University was Henry Schultz who was an econometrician and mathematical economist. He received both his B.A. (1936) and his Ph.D. (1943) in political science, from the University of Chicago.

In 1978 Simon received the Nobel Prize in Economic Sciences for "his pioneering research into the decision-making process within economic organizations". The announcement of the award also acknowledged his "contributions in, among other fields, science theory, applied mathematical statistics, operations analysis, economics, and business administration". Simon rejected what he saw as a common tendency to view society as a zero sum game. Simon argued: "There must be better games; and if I were to select a research problem without regard to scientific feasibility or interest, but only in terms of social welfare criteria, it would be the problem of finding conditions under which human beings will design and play games that all can win" (Simon, 1985).

Christopher Sims (1942–) earned his B.A. in *mathematics* from Harvard University *magna cum laude* in 1963 and his Ph.D. in economics from Harvard in 1968. Sims has published numerous important papers in his areas of research: econometrics and macroeconomic theory and policy. Among other things, he was one of the main promoters of the use of vector autoregression in empirical macroeconomics. He has also advocated Bayesian statistics, arguing for its power in formulating and evaluating economic policies.

Together with Thomas Sargent, he won the Nobel Prize in Economic Sciences in 2011 for their "empirical research on cause and effect in the macroeconomy" (http://en.wikipedia.org).

Vernon Smith (1927–) majored in electrical engineering, and during his senior year took an economics course. Intrigued by the similarity of economics to physics and engineering, Smith went to Kansas University for his master's degree in economics. He reflected on his switch to economics: "When I was a senior at CalTech, I took an economics course and I got interested in the topic. I went to the library, and among other things, I ran across a copy of Paul Samuelson's *Foundations of Economic Analysis*. It looked to me a lot like physics, which I was already doing. I also subscribed to the *Quarterly Journal of Economics*, and in one of the first issues there was an article by Hollis Chenery on engineering production functions - so it was engineering too!" (Smith, 2003).

In 2002, along with Daniel Kahneman, Smith won the Nobel Prize in Economics for "using laboratory experiments in empirical economic analysis and in the study of alternative market mechanisms" (Klein *et al.*, 2013).

Michael Spence (1943–) went to Princeton University for his undergraduate education. He earned his master's degree in *mathematics* from Oxford University and his Ph.D. in economics from Harvard. At Harvard, he studied under Kenneth Arrow and Thomas Schelling, among others.

In 2001, he was awarded the Nobel Prize in economics along with George Akerlof and Joseph Stiglitz "for their work on asymmetric information". Spence developed a theory of signaling; for example, individuals may attain educational distinctions to signal their productivity (Klein *et al.*, 2013).

Jan Tinbergen (1903–1994) studied mathematics and physics at the University of Leiden under Paul Ehrenfest (between 1921 and 1925). During those years at Leiden he had numerous discussions with Ehrenfest, Kamerlingh Onnes, Hendrik Lorentz, Pieter Zeeman, and Albert Einstein. After graduating, Tinbergen fulfilled his community service in the administration of a prison in Rotterdam and at the Central Bureau of Statistics (CBS) in The Hague. He then returned to the University of Leiden and in 1929 defended his Ph.D. Thesis titled (English) "Minimisation problems in Physics and Economics". This topic was suggested by Ehrenfest and allowed Tinbergen to combine *his interests in mathematics*, physics, economics and politics. At that time, CBS established a new department of business surveys and mathematical statistics, and Tinbergen became its first chairman, working at CBS until 1945. Access to the vast CBS data helped Tinbergen in testing his theoretical models. In parallel, starting from 1931 he was professor of statistics at the University of Amsterdam, and in 1933 he was appointed associate professor of mathematics and statistics at The Netherlands School of Economics, Rotterdam, where he stayed until 1973.

He was awarded the first Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel in 1969, which he shared with

Ragnar Frisch for" having developed and applied dynamic models for the analysis of economic processes" (http://en.wikipedia.org).

William Vickrey (1914–1996) went to Yale for his bachelor's degree and graduated in 1935 with a B.S. in *mathematics*. He received his master's in 1937 and Ph.D. in 1948 from Columbia University; being "a Quaker, a pacifist, and a moral economist" (Holt *et al.,* 1998: 1), Vickrey was a conscientious objector in World War II (Brownlee & Ide, 2013: 202). He became a professor at Columbia and remained there for the rest of his career.

In 1996, Vickrey was awarded the Nobel along with James Mirrlees "for their fundamental contributions to the economic theory of incentives under asymmetric information", in Vickrey's case, particularly for his work on taxation (Klein *et al.*, 2013).

Oliver Williamson (1932–), *attracted to math* and science in high school, decided to become an engineer. He enrolled in Ripon College's joint program with MIT and earned his bachelor's degree; while at MIT, Williamson was mentored by Kenneth Arrow. After working as an engineer, which gave him the "opportunity to learn about both big government and big business" and "got a good idea of how bureaucracy functioned" (Williamson, 1986), Williamson went to Stanford for graduate school, but decided to switch from business to economics (Williamson, 2010), moving on to Carnegie-Mellon for his Ph.D.

Focusing on the effect of formal and informal institutions on transaction costs, Williamson was a pioneer in what has become known as New Institutional Economics. In 2009, he won the Nobel Prize in Economics "for his analysis of economic governance, especially the boundaries of the firm," sharing the prize with Elinor Ostrom (Klein *et al.*, 2013).

4. Conclusions

After this short insight in the work of these distinguished economists, one can easily figure out the role of a solid mathematical background in analyzing, formulating and further studying economic problems or phenomena. Mathematics provides the right tools for

all the steps of this process, and this makes mathematics actually an intrinsic part of any economic scientific approach.

One of the aims of a scientific approach is to forecast the future development of a certain phenomenon and the mathematical models used in economics can do a reasonably good job of predicting some economic events. They also can provide information about when it is not possible to make reliable forecasts and, in addition, it is possible to outline a research that would improve the science of economics.

Among the research topics in economics that involve a considerable amount of mathematics let us shortly analyze an example meant to illustrate our point of view, the case of *mathematical finance*. Mathematical finance presents a special challenge, since it is a subject that is inherently interdisciplinary. Mathematical finance alias financial engineering is a subject that has more and more fans these days. Moreover, it has retained its popularity in the aftermath of the 2008 financial crises. Typically a subject restricted to graduate students, it is gradually diffusing its way into the undergraduate curriculum. This is in keeping with a long tradition in mathematics: for example, abstract algebra (groups, rings, fields, etc.) was taught exclusively to graduate students as recently as the 1940s, and is now taught to juniors in high schools.

The subject of mathematical finance, in order to help the economist students and researchers, involves multiple branches of mathematics, and some profound knowledge of economics, both concrete and intuitive. It also involves ideas that are completely new and are not present in other forms of mathematics. The topics in mathematics that one needs to know, or at least be familiar with, are basic mathematical analysis, probability, statistics, partial differential equations including free boundary problems, Monte Carlo methods, simulations, computer programming, Brownian motion, stochastic differential equations, and, on a more advanced level, topics such as numerical analysis, functional calculus and game theory. Given this, it is hard to imagine teaching such a subject with so many prerequisites to undergraduates, especially since most of the demand is not from students whose principal subject is mathematics.

In the present, more and more people discuss about the global economic crisis and his causes and effects. Economic science is frequently considered less viable than the physical sciences. Modern and sophisticated mathematical models of the economy have been developed but their accuracy is questionable because there are too many variables to dill with. The present economic crisis is often blamed on an unwarranted faith in faulty mathematical models.

In their paper, Sergio M. Focardi and Frank J. Fabozzi (Focardi and Fabozzi, 2010) claim that the mathematical handling of economics has actually been reasonably successful and that models are not the cause behind the present crisis. They said that the science of economics does not study immutable laws of nature but the complex human artefacts that are our economies and our financial markets, artefacts that are designed to be largely uncertain. We could make our economies and our markets less subject to uncertainty, and mathematical models more faithful to empirical data by introducing more rules and collecting more data. Collectively, the economists decided not to do so and therefore, models can only be moderately accurate. Still, the mathematical models offer a valuable design tool to engineer the economic systems. But the mathematics of economics and finance cannot be that of physics. The mathematics of economics and finance is the mathematics of learning and complexity, similar to the mathematics used in studies of biological or ecological systems. As we can see in paragraph three, most of the Nobel Prize in economics was awarded for innovation in game theory, applied mathematical statistics, analysis and analytical methods into economic theory, time series and creation of the econometrics models, in which the needs to use mathematical tools are inevitable.

Paul Krugman, one of the Nobel laureates, says he found himself in the science fiction of Isaac Asimov, especially the "Foundation" series - "It was nerds saving civilization, quants who had a theory of society, people writing equations on a blackboard, saying, 'See, unless you follow this formula, the empire will fail and be followed by a thousand years of barbarism'."... Social science, he says, offered the promise of what he dreamed of in science fiction - "the beauty of pushing a button

to solve problems. Sometimes there really are simple solutions: you really can have a grand idea" (Thomas, 2009).

Finally, we can say that mathematics could be the button to unleash the power of economy.

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